Short Papers

Efficient Computation of the Steady-State Response of Periodic Nonlinear Microwave Circuits Using a Convolution-Based Sample-Balance Technique

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Abstract—**This paper describes an efficient and robust approach to the computation of the steady-state response of periodic nonlinear microwave circuits. The problem of solving a set of differential equations, in this case, is converted into that of solving a system of nonlinear algebraic equations using a technique which is termed convolution-based sample balance. Although exact in all cases for which harmonic-balance techniques are exact, this technique does not require the use of discrete Fourier transforms, and calculating the Jacobian is straightforward. For the solution of the resulting system of nonlinear equations, an efficient and yet very robust algorithm has been developed. In the examples given, savings in computational effort of over 85% are reported when this algorithm is compared with Newton's method.**

I. Introduction

Harmonic-balance (HB) techniques are probably the most widely used techniques for calculating the steady-state response of nonlinear microwave circuits. They are based on transforming the problem of solving a set of differential equations into that of solving a system of nonlinear algebraic equations. A key factor for the successful implementation of HB techniques is therefore the algorithm used to solve the system of equations, and several relatively fast relaxation algorithms have been used for this purpose. However, it is generally agreed that robust, general-purpose algorithms should use Newton's method or its variations, either directly solving the system of equations or solving a related optimization problem [1], [2].

Sample-balance (SB) techniques [1] differ from HB techniques by expressing the state variables in the time domain rather than in the frequency domain. The two state variable representations, however, are equivalent since one is the Fourier transform of the other. The convolution-based sample-balance (CBSB) technique is therefore directly related to HB techniques, but its formulation is simpler, especially when the Jacobian is required. Contrary to other SB techniques [1], [3], neither base functions nor discrete Fourier transforms are needed.

For the solution of the resulting system of nonlinear equations, an efficient algorithm has been developed. Although it is a general-purpose algorithm for the solution of systems of nonlinear equations, it is particularly suitable for the kind of systems that arise when HB or SB techniques are applied to the solution of nonlinear circuits. The performance of the CBSB technique together with this algorithm is demonstrated through the simulation of a Schottky diode and a MESFET. The combination results in excellent convergence properties, and considerable savings are achieved by this algorithm when compared with Newton's method, with no loss of robustness. Results show over 85% reduction in computational effort. In some situations, further savings are also possible by not calculating all the columns of the Jacobian and by using interpolation. The computational effort does not change even in highly nonlinear circuits and this permits the approach described in this work to handle a wide range of situations efficiently.

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II. The Convolution-Based Sample-Balance Technique

The CBSB technique is a SB technique [1] in which an appropriate convolution theorem is used to bypass the need to work in both the frequency and the time domain. In different contexts, convolution integrals are used as aids in the solution of nonlinear circuits; for instance, they are used in [4] with a time-domain method and in [5] with an HB method.

As in piecewise HB techniques, the circuit to be analyzed is initially divided into two parts, generally called linear and nonlinear subcircuits (Fig. 1(a)). The only restriction is that the linear part contains only linear circuit elements and the division is assumed to be made through P ports. The equivalent circuit of Fig. 1(b) can be found for each port at frequencies $\omega_m = m\omega_0$, $m = 0, 1, \cdots, M$. This is done using Norton's theorem and the superposition theorem, and $[Y^{rs}(\omega_m)]$ is the Y matrix of the linear part with all sources turned off at frequency ω_m; the $U^r(\omega_m)$'s are the Norton current sources. In steady-state, the currents through the linear elements in Fig. 1(b) (admittances and voltage-controlled current sources) have the form

$$i(t) = \sum_{m=-M}^{M} Y(\omega_m) V(\omega_m) \exp(j\omega_m t). \tag{1}$$

If $T = 2\pi/\omega_0$, the convolution theorem for periodic signals [6] can be used to express $i(t)$ in terms of time-domain quantities:

$$i(t) = \frac{1}{T}\int_0^T v(t-\tau)y'(\tau)\,d\tau \tag{2}$$

where

$$y'(t) = \sum_{m=-M}^{M} Y(\omega_m)\exp(j\omega_m t) \tag{3}$$

can be recognized as the response (truncated to frequency ω_M) to an impulse train with frequency ω_0, although this is irrelevant since (3) provides a simple way of calculating $y'(t)$. For both $v(t)$ and $y'(t)$ sampled at N evenly distributed time samples and $t_n = nT/N$, (2) can be discretized as

$$i(t_n) = \frac{1}{N}\sum_{k=0}^{N-1} v(t_{n-k})y'(t_k) = \sum_{k=0}^{N-1} v(t_{n-k})y(t_k) \tag{4}$$

where $y(t_n) = y'(t_n)/N$. The fundamental characteristic of (4) is that it is **exact** if both $v(t)$ and $y'(t)$ are sampled above the Nyquist rate. This is a direct consequence of the convolution theorem for discrete Fourier transform pairs [6] and means that the CBSB technique is exact in all cases for which HB techniques are exact. This condition requires that $M < N/2$. Another important aspect is that the $y(t_n)$'s are constants and need be computed only once, before the simulation starts.

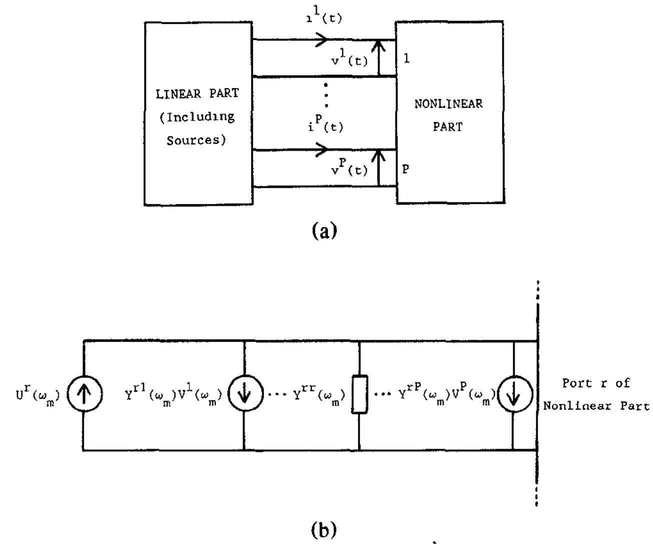

Fig. 1. (a) Circuit partitioning. (b) Equivalent circuit of linear part at port r and frequency ω_m.

If Kirchhoff's current law is applied at each port and instant in time, t_n, $n = 0,\cdots,N-1$, the discretized problem can be conveniently written with the aid of (4) as

$$\begin{bmatrix} y^{11} & y^{12} & \cdots & y^{1P} \\ \vdots & \vdots & & \vdots \\ y^{P1} & y^{P2} & \cdots & y^{PP} \end{bmatrix} \cdot \begin{bmatrix} v^1 \\ \vdots \\ v^P \end{bmatrix} + \begin{bmatrix} i^1 \\ \vdots \\ i^P \end{bmatrix} - \begin{bmatrix} u^1 \\ \vdots \\ u^P \end{bmatrix} = 0 \quad (5)$$

where, using (4), y^{rs} is a Toeplitz matrix defined as

$$y^{rs} = \begin{bmatrix} y_0^{rs} & y_{N-1}^{rs} & \cdots & y_1^{rs} \\ \vdots & \vdots & & \vdots \\ y_{N-1}^{rs} & y_{N-2}^{rs} & \cdots & y_0^{rs} \end{bmatrix} \quad (6)$$

with $y_n^{rs} = y^{rs}(t_n)$. The v^r's, i^r's, and u^r's are N-dimensional vectors representing respectively port voltages, port currents, and source currents. For instance, if $v_n^r = v^r(t_n)$,

$$v^r = (v_0^r,\cdots,v_{N-1}^r)^T. \quad (7)$$

If the variables that control voltages and currents at the ports of the nonlinear subcircuit are stacked in a PN-dimensional vector x, (5) can be finally expressed in a compact form, with the obvious definitions, as

$$F(x) = y \cdot v(x) + i(x) - u = 0. \quad (8)$$

Equation (8) defines a system of nonlinear equations of order PN. In HB techniques, the equivalent of (8) involves frequency-domain as well as time-domain quantities since nonlinear circuits are invariably described in the time domain. Here, only time-domain variables and expressions are present.

Y matrices may be restrictive with respect to the kind of circuit that can be analyzed since they are not always well defined. In such cases, similar formulations can be developed using other descriptions of the linear circuit (Z matrices, S matrices, etc), although it was found that Y matrices are able to handle most of the usual situations.

III. The Algorithm for Solving the System of Nonlinear Equations

Quasi-Newton or modification methods [7] have recently been considered for microwave applications [9], [10] and when compared with Newton's method, they require much less computational effort per iteration but have a slower rate of convergence. Systems of nonlinear equations can be viewed as n-dimensional functions for which n-dimensional roots are required. Since the goal is to find the root with minimum effort, which method performs best depends on the particular problem being solved. The usual measure of computational effort is the number of times the function is evaluated in the process of finding its root. This includes function evaluations required to calculate derivatives numerically. In particular, evaluating $v(x)$ and $i(x)$ in (8) is usually time consuming, especially if physical models are used.

Reference [11] provides useful practical information on methods for solving systems of nonlinear equations. Several different algorithms are compared in a number of different problems. Algorithms based on quasi-Newton methods were found to perform better than those based on Newton's method in problems in which either the number of equations is large or the Jacobian is expensive to calculate. Since these conditions are inevitably present in (8), it was decided to use a quasi-Newton method.

The most representative and well known quasi-Newton method is probably Broyden's method [12] and its performance can be improved if it is used with projected updates [13]. Broyden's method with projected updates forms the basis of the algorithm described in this section and is termed the modified Broyden's method.

Quasi-Newton methods have the general form [7]

$$x^{k+1} = x^k - A^k \cdot F(x^k) \quad (9)$$

where A^k is a matrix that is expected to be close to $J^{-1}(x^k)$ (the inverse of the Jacobian at x^k) in some norm. One problem with quasi-Newton methods is that, at each iteration, A^k is not updated in directions orthogonal to $h^k = x^{k+1} - x^k$ (the step taken). This problem can eventually lead to the failure of the method. One way of avoiding this problem is described in [14] and consists in taking occasional steps in directions linearly independent of the last h^k's. These linearly independent steps allow A^k to resemble the inverse of the Jacobian in all directions and have been used along with the original Broyden method in the context of optimization [9]. A key issue regarding overall efficiency is when to take these linearly independent steps. The modified Broyden method provides a built-in criteria for the application of linearly independent steps. When the last h^k's are not sufficiently linearly independent, the method goes through what is called a restart in [13], and a linearly independent step is therefore taken whenever the method restarts. The directions of the linearly independent steps are generated as described in [14].

Fig. 2 is a representation of the algorithm. For additional robustness, a Newton iteration is attempted when the modified Broyden method fails to reduce the norm of the function more than L_m consecutive times. L_m is usually set to 1 or 2. In order to avoid excessively large step sizes, which can lead to failure of any iterative method, the maximum norm of h^k is limited [8].

A subroutine, SOLVNL, was developed based on the algorithm of Fig. 2 which can also implement Newton's method with minor modifications. SOLVNL has been tested and compared with Newton's method for several problems [11]–[13]. The results were excellent with respect to both efficiency and ability to find a solution starting from a poor estimate of the root.

IV. Approximate Evaluation of the Jacobian Through the Use of Interpolation

The derivatives needed to evaluate the Jacobian J of $F(x)$ in (8) can be calculated either numerically, or, when possible, analytically. In this work, a numerical approach has been adopted for reasons similar to those discussed in [10] and because the use of physical models would make the analytical approach cumbersome if not impossible. The numerical approach adopted

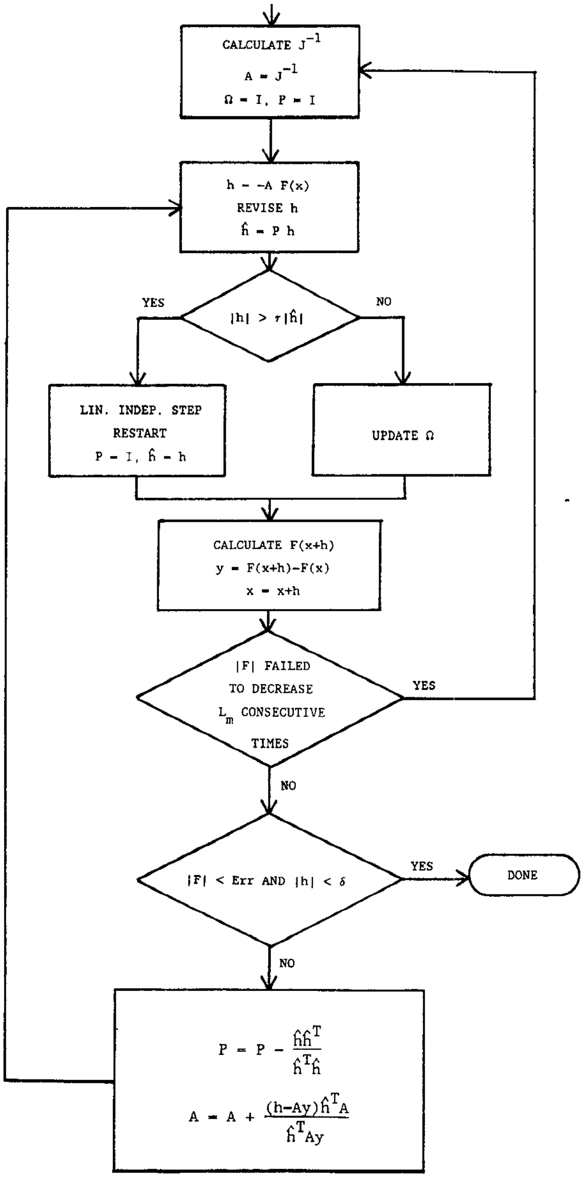

Fig. 2. Diagram representing the algorithm of subroutine SOLVNL.

avoids unnecessary calculations and is not much slower than a direct analytical evaluation of the derivatives, although the analytical approach is better with respect to accuracy. In general, an n-dimensional system of equations requires $n+1$ function evaluations to calculate $F(x)$ and J numerically. In this case, treating the nonlinear devices in the nonlinear part individually greatly simplifies this task. The Jacobian of $F(x)$ is given by

$$J = y \cdot \frac{\partial v}{\partial x} + \frac{\partial i}{\partial x} \tag{10}$$

where

$$\frac{\partial i}{\partial x} = \begin{bmatrix} \frac{\partial i^1}{\partial x^1} & \cdots & \frac{\partial i^1}{\partial x^P} \\ \vdots & & \vdots \\ \frac{\partial i^P}{\partial x^1} & \cdots & \frac{\partial i^P}{\partial x^P} \end{bmatrix} \tag{11}$$

and

$$\frac{\partial i^r}{\partial x^s} = \begin{bmatrix} \frac{\partial i_0^r}{\partial x_0^s} & \cdots & \frac{\partial i_0^r}{\partial x_{N-1}^s} \\ \vdots & & \vdots \\ \frac{\partial i_{N-1}^r}{\partial x_0^s} & \cdots & \frac{\partial i_{N-1}^r}{\partial x_{N-1}^s} \end{bmatrix}. \tag{12}$$

The quantity $\partial v/\partial x$ is defined in a similar way. All blocks defined by (12) can be calculated numerically by evaluating $F(x)$ once and simulating each individual nonlinear device an extra pN times, where p is the number of device ports. This can be much less than evaluating $F(x)$ in (8) PN times. A useful property of (10), which is used in Section V with the algorithm of Section III and Newton's method, is that J does not depend on the sources.

The elements of the $N \times N$ matrices defined by (12) are not completely independent. Because of their physical meaning, $\partial i_{j+k}/\partial x_j$ should be a well-behaved function of j for fixed k when N is large; it is expected, therefore, that this function can be interpolated in j reasonably well. Based on this observation, only some of the columns of (12) are calculated while the remaining elements are interpolated diagonally. The interpolation has to be performed in a wraparound fashion if either column 0 or $N-1$ is not computed.

In order to make this strategy more effective, it was implemented in a manner such that the columns evaluated and those interpolated changed in a cyclic fashion every time the Jacobian was required. In preliminary tests, both spline and linear interpolation were tried and the latter, since it showed more reliable performance, was subsequently adopted. The distance between columns actually calculated is denoted by IS in the next Section. If $IS = 1$, no interpolation is used.

V. Results

A general-purpose computer program, NLCKT, has been developed using the CBSB technique and the subroutine SOLVNL and this is general enough for equivalent-circuit as well as physical models for a variety of devices to be included. Derivatives are calculated using second-order backward finite-difference expressions, which provide sufficient accuracy at $N =$ 8 or greater. Higher order expressions can be added if needed and time delays (as those between gate and drain of MESFETs) can be easily handled with the aid of interpolation. The Jacobian can be approximated as described in Section IV by setting the parameter IS. In order to demonstrate the capabilities of NLCKT, a Schottky diode and a MESFET model developed by other authors were simulated. No attempt to discuss the accuracy of the results is made, and this is mainly a characteristic of the models rather than of the methods of circuit analysis; instead, attention is concentrated on the numerical performance achieved. These two models provide a good test for NLCKT as both include exponential diodes and nonlinear capacitances.

In all cases, the solution of the nonlinear system of equations that results from using the CBSB technique is carried out using the subroutine SOLVNL and, for comparison, Newton's method with the same parameters being used. Also, in problems in which the source voltage varies, the solution and the last Jacobian (see Section IV) of the previous source voltage are used as initial values. The convergence criteria adopted throughout are $|F| < \text{Err} = 10^{-5}$ A and $|h| < \delta = 10^{-4}$ V (Fig. 2), and Euclidean norm is used.

A. Schottky Diode Simulation

The input impedance of the model of the Alpha Industries diode DMC5504C-075 given in [15] was calculated in a 50 Ω system. Fig. 3 shows the circuit analyzed and the parameter values used. As a test for robustness, the initial values were first

TABLE I
NUMBER OF FUNCTION EVALUATIONS REQUIRED TO REACH THE SOLUTION IN SEVERAL CASES

	NEWTON			SOLVNL		
	$IS=1$	$IS=2$	$IS=3$	$IS=1$	$IS=2$	$IS=3$
(i)	137	100	79	63	65	53
(ii) $N=16$	1164	1305	1586	212*	182	223
(ii) $N=32$	2204	1740	1462	258	242	237

*0.6 s of CPU time on an Amdahl V7 computer.

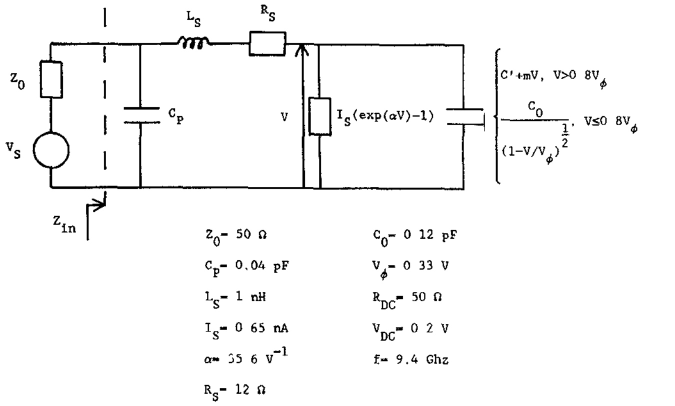

Fig. 3. Model of the Schottky diode simulated and parameter values used.

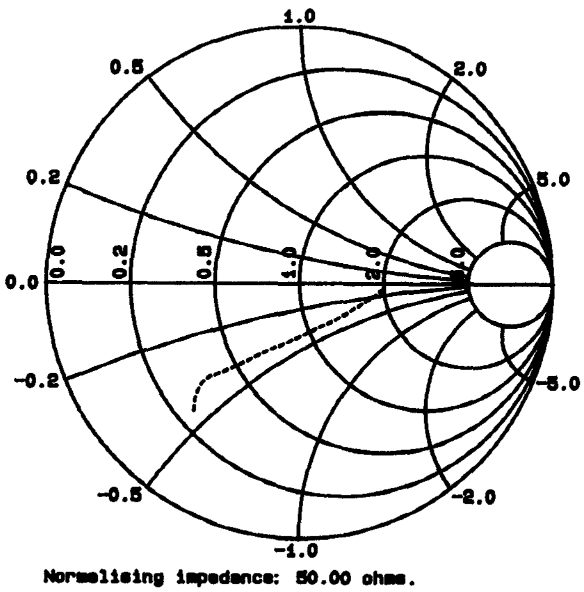

Fig. 4. Input impedance of the Schottky diode simulated for source voltages from 0.1 to 3 V. The direction of increasing power is from left to right.

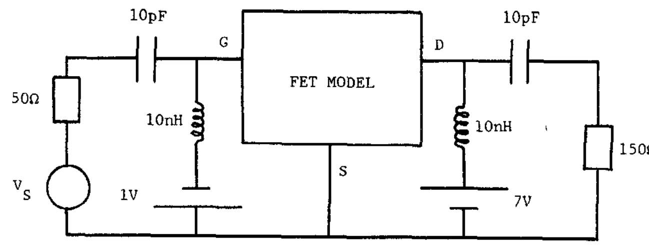

Fig. 5. MESFET circuit simulated.

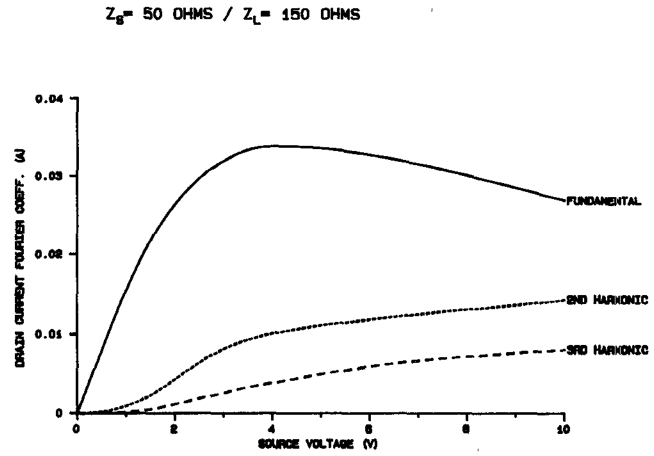

Fig. 6. Fourier coefficients at fundamental frequency and second and third harmonics of the drain current of the MESFET simulated as a function of source voltage.

set to their dc condition and the source voltage to 1 V. The number of function evaluations required to reach the solution for $N=16$ is shown in the first row of Table 1 for several cases. The improvements achieved by the techniques described in this paper are considerable.

The source voltage was then varied from 0.1 to 3 V in 0.1 V steps. Results for $N=16$ and $N=32$ are also displayed in Table I while the input impedance is given in Fig. 4. The improvements achieved over Newton's method are dramatic. It is also clear that interpolating the Jacobian is more effective for $N=32$. The result for $IS=2$ and $N=16$, which corresponds to savings of over 83% when compared with Newton's method, is remarkable since an average of six function evaluations were required to solve 30 systems of equations of order 16. Similar comments do not apply for $N=32$ since, in this case, the problem is clearly oversampled.

B. MESFET Simulation

The MESFET model simulated is similar to that described in [16]. The main difference is the expression for the drain current, which was simplified to [17]

$$i_d(v_g, v_d) = \beta(v_g + V_T)^2(1 + \lambda v_d)\tanh(\alpha v_d). \quad (13)$$

The saturation characteristics for a MESFET with parameters given in [16] and for $\beta = 7.5$ mA/V^2, $V_T = 2.5$ V, $\lambda = 0.01$ V^{-1}, and $\alpha = 1.8$ V^{-1} were calculated. The MESFET is embedded in the circuit of Fig. 5 and the frequency is 10 GHz. N was set to 16, IS to 1, and the source voltage was varied from 0 to 10 V in a total of 30 evenly spaced points. A total of 2930 function evaluations were required by Newton's method for the whole task while SOLVNL needed only 314, corresponding to savings of 89%. The absolute value of the Fourier coefficients of the first three harmonics of the drain current are shown in Fig. 6 as a function of the source voltage. The source voltage was deliberately increased to unreasonable values to test the ability of the program to cope with highly nonlinear situations, but even so the number of function evaluations did not change significantly with source voltage. (Similar tests performed with the diode model led to the same conclusion.) This is in part due to the relative insensitivity to nonlinearities of the structure of the Jacobian in the CBSB technique, whereas, in contrast, Jacobians in HB techniques change considerably in large-signal situations. Also, the variables in SB techniques are automatically scaled to the same level while in HB techniques the levels at the higher frequencies considered should be orders of magnitude smaller than the levels at the first few harmonics. Other tests confirmed the excellent performance and robustness of the program NLCKT.

VI. Comments on Extensions of the Techniques Described in This Work

The CBSB technique can be extended to multitone analysis by employing multidimensional [10] or quasi-periodic [18] Fourier transforms since convolution theorems exist for both of these [6], [19]. As both approaches rely on calculating derivatives in the frequency domain, however, an extra convolution would have to be performed.

Results from Section V suggest that the program NLCKT is particularly efficient when a good estimate of the solution is available. This indicates that NLCKT should be suitable for use with continuation methods [20].

The subroutine SOLVNL can be modified to handle situations in which the Jacobian is ill-conditioned by using the ideas described in [14]. This would further increase the range of applicability of the approach described in this work.

The superiority of quasi-Newton methods over Newton's method for problems in which either the Jacobian is expensive to calculate or the number of variables is large was established in [11] and has been substantiated in circuit analysis applications by the results in Section V. However, one possibility that has not been investigated is the effect of having a sparse Jacobian. For physical models, the computation time is dominated by computing $F(x)$ and J but, for equivalent-circuit models, the computation time can become increasingly dominated by matrix handling as the number of variables increases so that sparsity of the Jacobian could then become a major factor. In piecewise circuit-analysis techniques, the Jacobian is not sparse although, by setting some elements to zero based on a physical criteria, sparse-matrix techniques can be exploited in multitone problems involving a large number of frequencies [21], [22]. In such problems, this is certainly advantageous with respect to memory space required. For an assessment of how this would change the conclusions drawn in [11], the slowdown in the rate of convergence caused by the use of an approximate Jacobian would have to be known. As power levels increase, this slowdown eventually degenerates into divergence, leading to the concept of dynamic range of the sparse-matrix analysis mentioned in [21]. This discussion can be illustrated by looking at the problem examined in [21] and [22], which involves $n = 3600$ variables. The computation time per iteration of the matrix handling required in quasi-Newton methods is proportional to n^2 while in Newton's method it is proportional to $1/3n^3$ [8]. In this problem, quasi-Newton methods require 1200 times less computational effort in matrix handling per iteration while savings of 550 times are claimed in [22]. The times given in [21] indicate that, with the use of sparse-matrix techniques, less than 1/30 of the total computational time is spent in matrix handling and therefore Jacobian evaluation becomes the most demanding task. Assuming that the Jacobian in this problem can be calculated analytically and that the expressions for the derivatives are not too complex, the Jacobian evaluation could be carried out "cheaply" in the sense used in [11]. Therefore, in this problem, Newton's method combined with sparse-matrix analysis should be superior to quasi-Newton methods if the use an approximate Jacobian does not significantly affect the rate of convergence.

VII. Conclusions

A new and efficient approach to computing the steady-state response of periodic nonlinear microwave circuits has been presented. It combines a technique that is termed convolution-based sample-balance with a robust algorithm for the solution of the resulting system of nonlinear equations. Because of the convolution-based sample-balance technique, this approach permits the analysis of highly nonlinear circuits without increasing the computational effort and the algorithm used can be almost an order of magnitude faster than Newton's method. It should therefore be considered as an alternative to the latter in other nonlinear circuit analysis applications. The excellent results which can be achieved by this approach were demonstrated by simulating a Sckottky diode and a MESFET.

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Phase Shift Determination of Imperfect Open Calibration Standards

Gary Biddle

Abstract —**A new measurement technique for determining the inherent phase shift of open calibration standards for network analyzers due to fringing capacitance is presented. The resultant phase shift is directly measured using an uncalibrated network analyzer and requires no modeling of coefficients of capacitance as conventional methods do. An exact expression for the phase shift of an imperfect open is derived for each frequency point. Two sets of standard one-port error equations are developed for the application. The traditional set of calibration standards, the match, short, and imperfect open, are used. The standards are measured twice: once at the reference plane and then offset by a precision piece of air line. Results are presented for the phase shifts of a few open calibration standards at discrete frequencies.**

I. Introduction

Network analyzers have been used extensively to characterize microwave components and devices for several decades. Improvements in instrumentation hardware, computer availability, and new calibration standards and procedures have enhanced measurement capabilities.

Initially, with the formulation of signal flow graphs well documented, early works by Hackborn [1] and Hand [2] introduced the automatic network analyzer system. Attention focused on hardware description, calibration procedures, and measurement accuracy. The error models appearing in these works were eight-term with the following calibration standards: the match, the short, and the offset short.

A few years later, an open calibration standard was introduced as an option to the offset short by Kruppa [3]. By 1978 it was known and pointed out in works by Rehnmark [4], daSilva and McPhun [5], and Fitzpatrick [6] that opens were imperfect because of radiation and stray capacitance.

The phase shift of an imperfect open was addressed by daSilva and McPhun. The measurement procedure required four test pieces with identical terminations, identical propagation factors for offsets of prescribed lengths, and a short circuit test piece. A total of five measurements were required.

Hewlett Packard approached the phase shift problem of an imperfect open in a different manner. In Application Note 221A, an accuracy enhancement program using coefficients of capacitance to correct for the residual fringing effects of a shielded open was presented. The resultant phase shift was modeled as a function of frequency. The coefficients of capacitance were then chosen to best fit the selected measurement responses.

In this paper, a new measurement technique that obtains the phase shift directly is presented. The conventional calibration standards are used: the match (fixed and/or sliding load), the short, and the imperfect open. One piece of precision air line is also required.

In contrast to prior measurement procedures, no special test pieces are needed, no identical terminations or propagation factors for prescribed lengths are required, and no coefficients of capacitance are required.

II. Error Equations

This section shows the two sets of error equations needed to determine the phase shift of the imperfect open. The conventional methods of signal flow analysis, using Mason's rule, are employed. The well-known one-port error network is obtained.

In order to determine the open's phase shift, a total of six measurements must be made. The first set of measurements require the three standards to be measured at a reference plane. The error terms of the reference plane are unknown. Thus it is an uncalibrated measurement.

The second set of measurements require the three standards to be measured again at the same reference plane but offset with a precision piece of air line. Again the error terms are unknown and the measurement is uncalibrated. The introduction of the air line into the second set of measurements has added an additional propagation factor which is unknown.

The reflection coefficients of the calibration standards and the propagation factor of the air line appear in the error network diagrams. The three reflection coefficients are treated as follows.

The match in the ideal case is reflectionless, thus having a reflection coefficient of zero. The reflection coefficient of the match is represented as zero in the error equations. The standard practice of utilizing the sliding load at higher frequencies to enhance the measurement of the true system directivity is used.

The short in the ideal case reflects all the incident energy with a phase inversion at all frequencies. The reflection coefficient of the short is represented as $|1|$ with an argument of π in the error equations. The precision shorts found in 3.5 mm, 7 mm, and 14 mm calibration kits have very low residual inductance; thus they may be considered ideal for this measurement technique.

The open in the ideal case reflects all the incident energy with no phase shift. In practice, there is an appreciable phase shift associated with an open. The reflection coefficient of the imperfect open is represented as $|1|$ with an unknown argument in the error equations. Thus only four measurements can be made of known calibration standards.

The precision piece of air line is required to offset the calibration standards. The air line is considered to be reflectionless with unknown propagation factors and length. It is depicted as such.

The flow graphs for the two sets of error equations are shown in Fig. 1 and Fig. 2. The upper flow graph depicts the standard one-port error network. The lower flow graph depicts a one-port error network which includes an additional offset.

The variable Γ_m is the reflection coefficient measured at the analyzer's measurement plane, while Γ_{cs} represents the reflection coefficient of the calibration standard applied at the reference plane. In the conventional way, the three error terms $\exp(-kl)$ represents the offset introduced by the reflectionless air line.

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